Prognostics on Laser Systems

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Overview

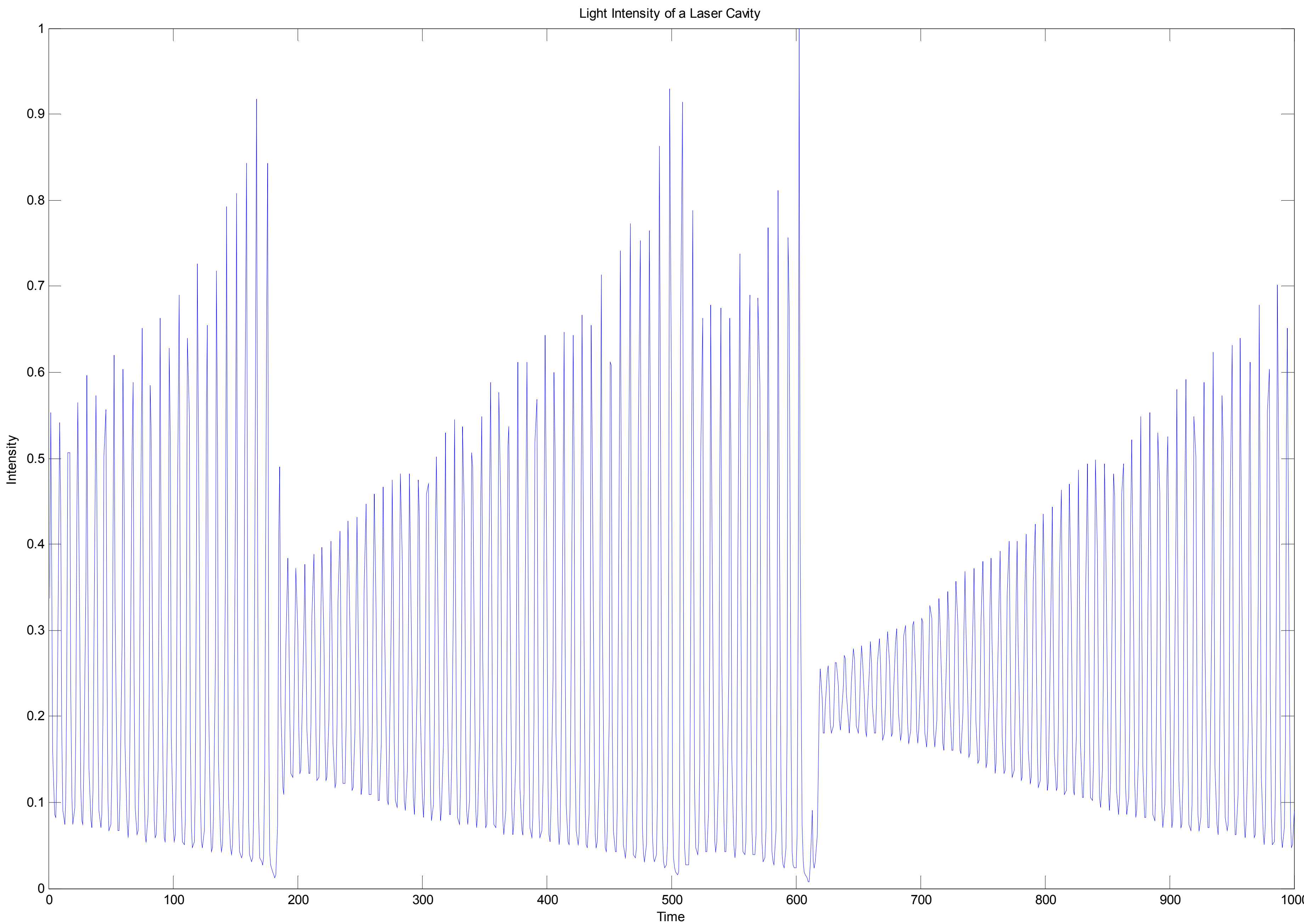
- This presentation discusses the development of a set of algorithms to do event precursor analysis on intensity data from a laser.
- The data is from a well-studied (NH_3) laser system that has chaotic behavior.

Laser Data I

- The laser undergoes periods of buildup of intensity followed by a sudden collapse in intensity.
- Sometimes the collapse is significant, and other times it is relatively small.
- It is hard to predict what type of collapse will occur (i.e., it is a chaotic process).

Laser Data II

- It is known that one can approximate the dynamical behavior of the laser using ideas from nonlinear dynamical systems.
- $dx/dt = s(y-x)$
  $dy/dt = rx - y - xz$
  $dz/dt = xy - bz$
- The values of s, r, and b determine the nature of the chaotic attractor.


Light Intensity of a Laser Cavity
Intensity
Time
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
100
200
300
400
500
600
700
800
900
1000

Prognostic Problem

- Given a small set of data (1000 points) develop an algorithm that can:
  - Predict the future dynamics of this system.
  - Generate a signal that represents the confidence in the prediction.

Method

- We address this problem using the theory of Gaussian Processes (which are related to Kernel Methods), which assumes that any subset of data for a vector X is Gaussian distributed (from wikipedia).

$$\vec{\mathbf{X}}_{t_1,\ldots,t_k} = (\mathbf{X}_{t_1}, \ldots, \mathbf{X}_{t_k})$$

Using characteristic functions of random variables, we can formulate the Gaussian property as follows: $\{X_t\}_{t \in T}$ is Gaussian if and only if for every finite set of indices $t_1, \ldots, t_k$ there are positive reals $\sigma_{l\,j}$ and reals μ_j such that

$$\mathrm{E}\left(\exp\left(i \sum_{\ell=1}^{k} t_\ell \, \mathbf{X}_{t_\ell}\right)\right) = \exp\left(-\frac{1}{2} \sum_{\ell,j} \sigma_{\ell j} t_\ell t_j + i \sum_{\ell} \mu_\ell t_\ell\right).$$

The numbers $\sigma_{l\,j}$ and μ_j can be shown to be the covariances and means of the variables in the process.

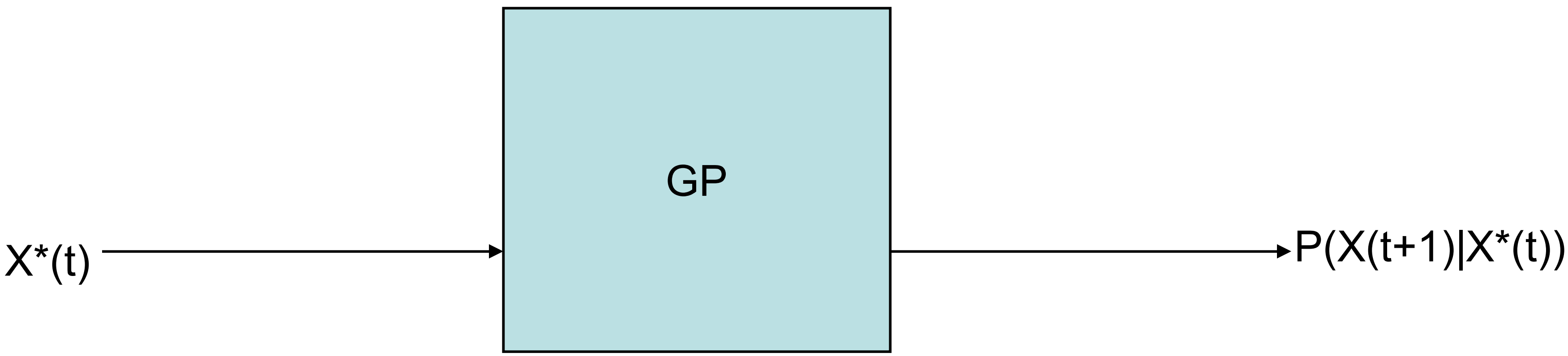
Approach

- Using delay coordinate embedding (and thus Takens' Theorem) we build a Gaussian Process Regression (GPR) to predict:

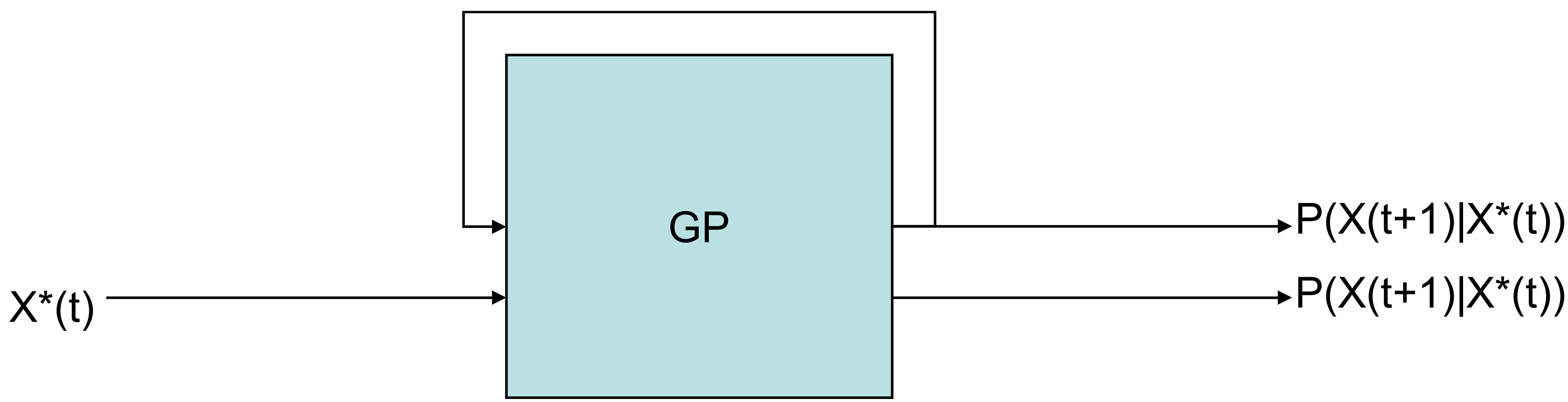
  $$P(X(t+1)|X(t), X(t-1), \ldots, X(t-d)) = P(X(t+1)|X^*(t))$$

- Once this distribution is known, we can make predictions through iterating the distribution.

One Step Ahead Predictions



Iterated Predictions



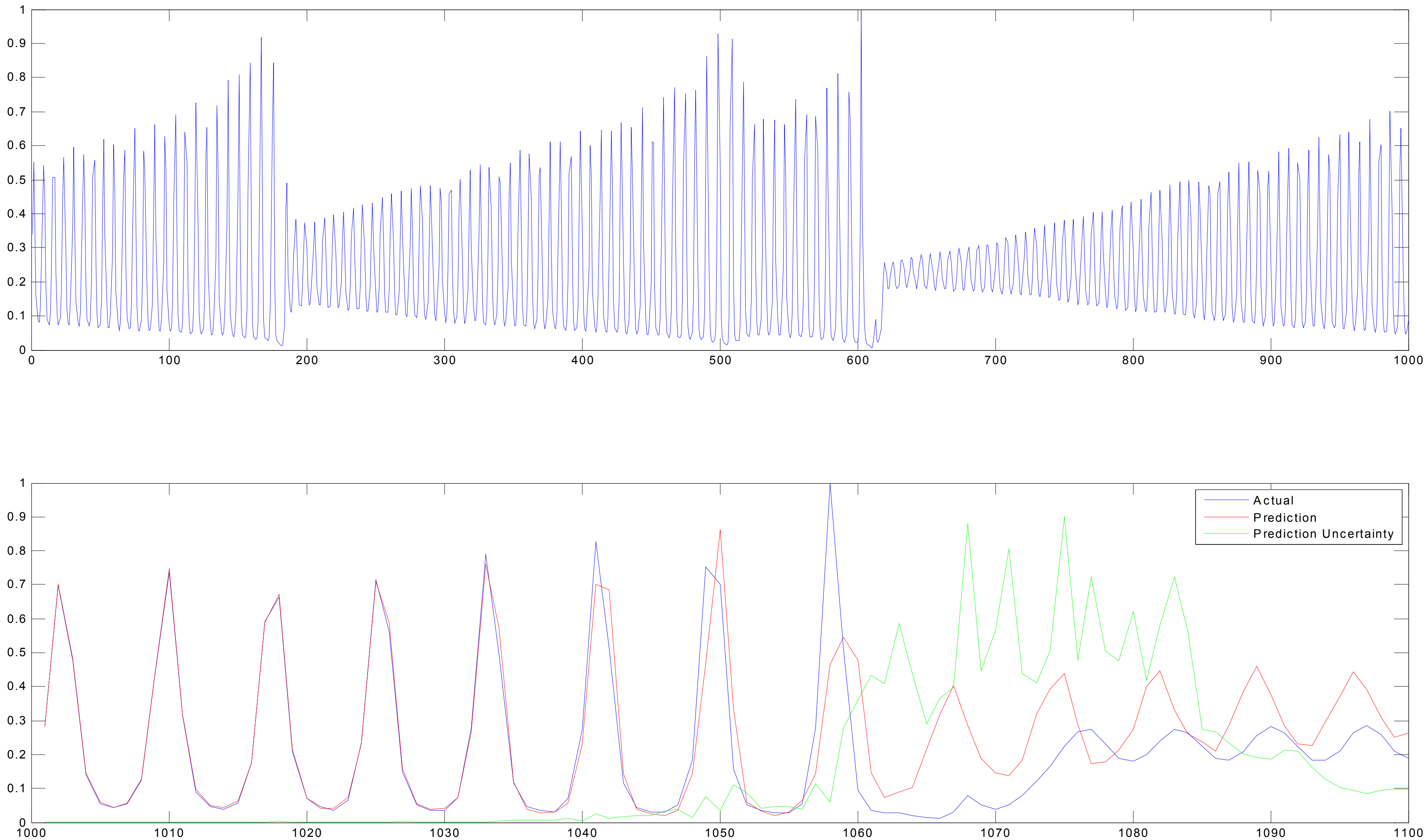
i.e., we feed the output of the model into its input to make a prediction of P(X(t+2) | [P(X(t+1), X(t), X(t-1), … X(t-d+1])

100-step ahead forecasts

- We iterate the Gaussian Process 100 times to generate this time series.

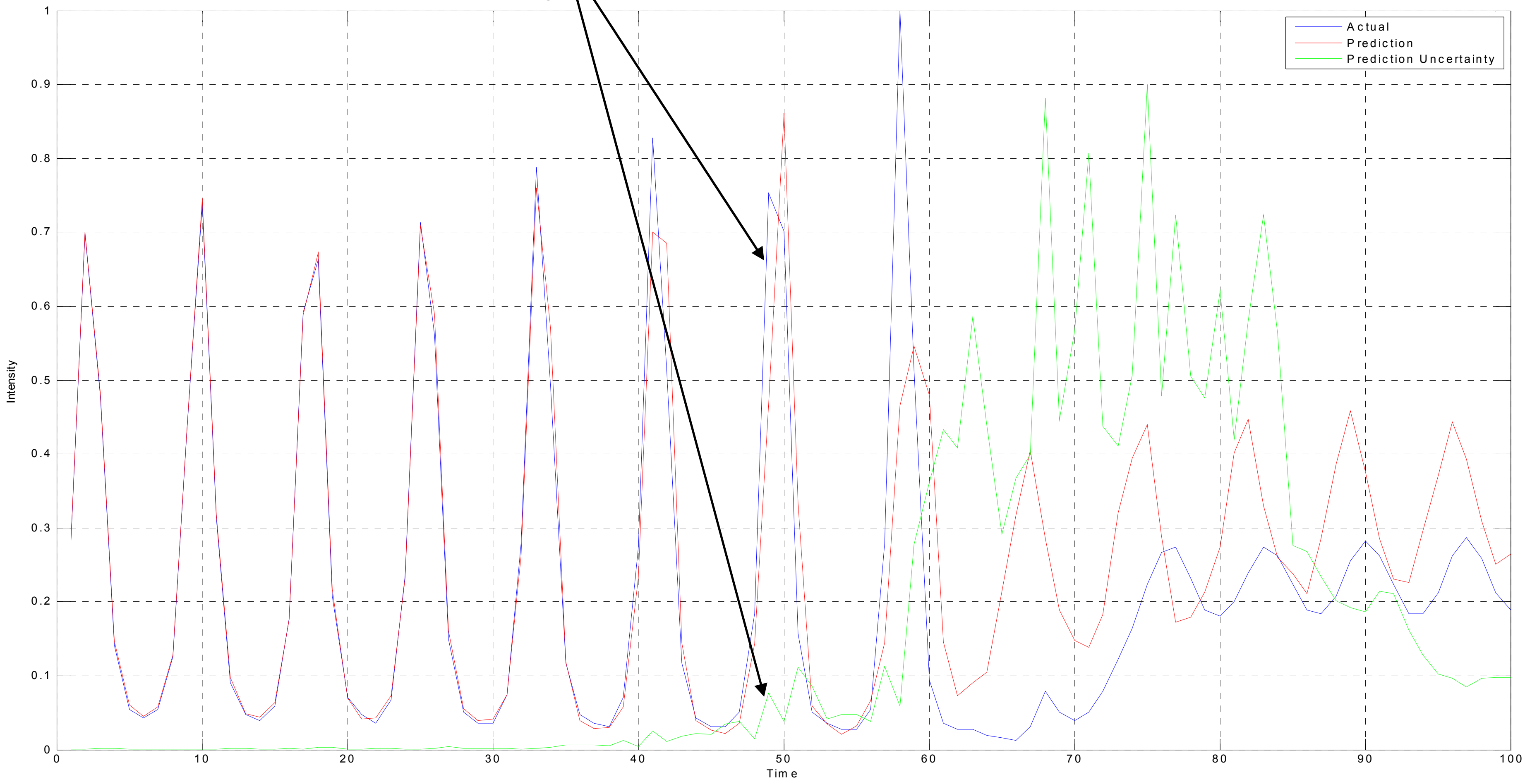
- Forecasting metric: normalized mean squared error.
  - Trial A: 0.30
  - Trial B: 0.16

GP Trial A: NMSE = 0.30

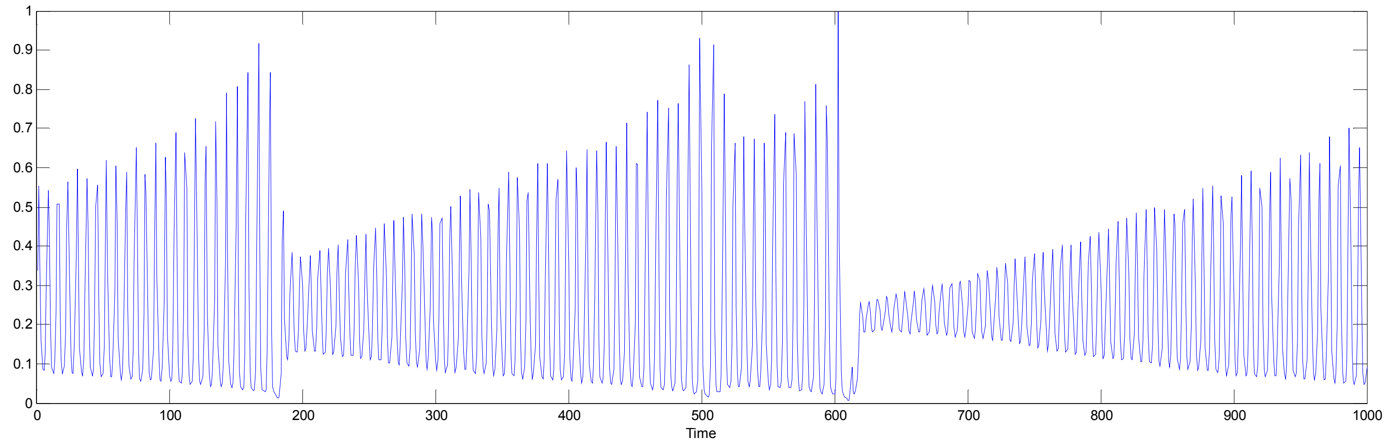


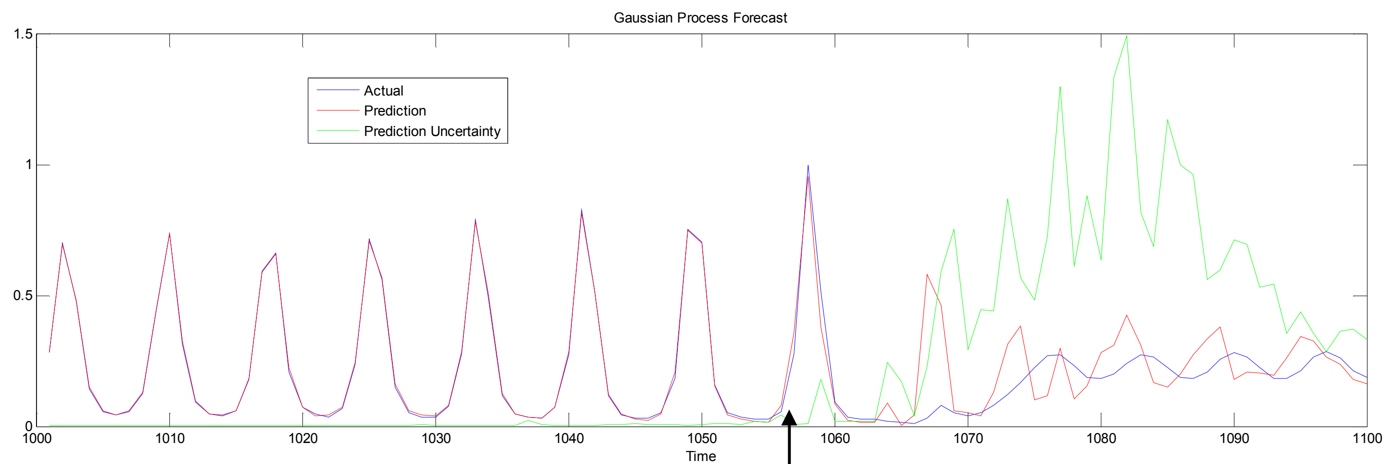
GP Trial A: NMSE = 0.30

Actual error increases as does the uncertainty.



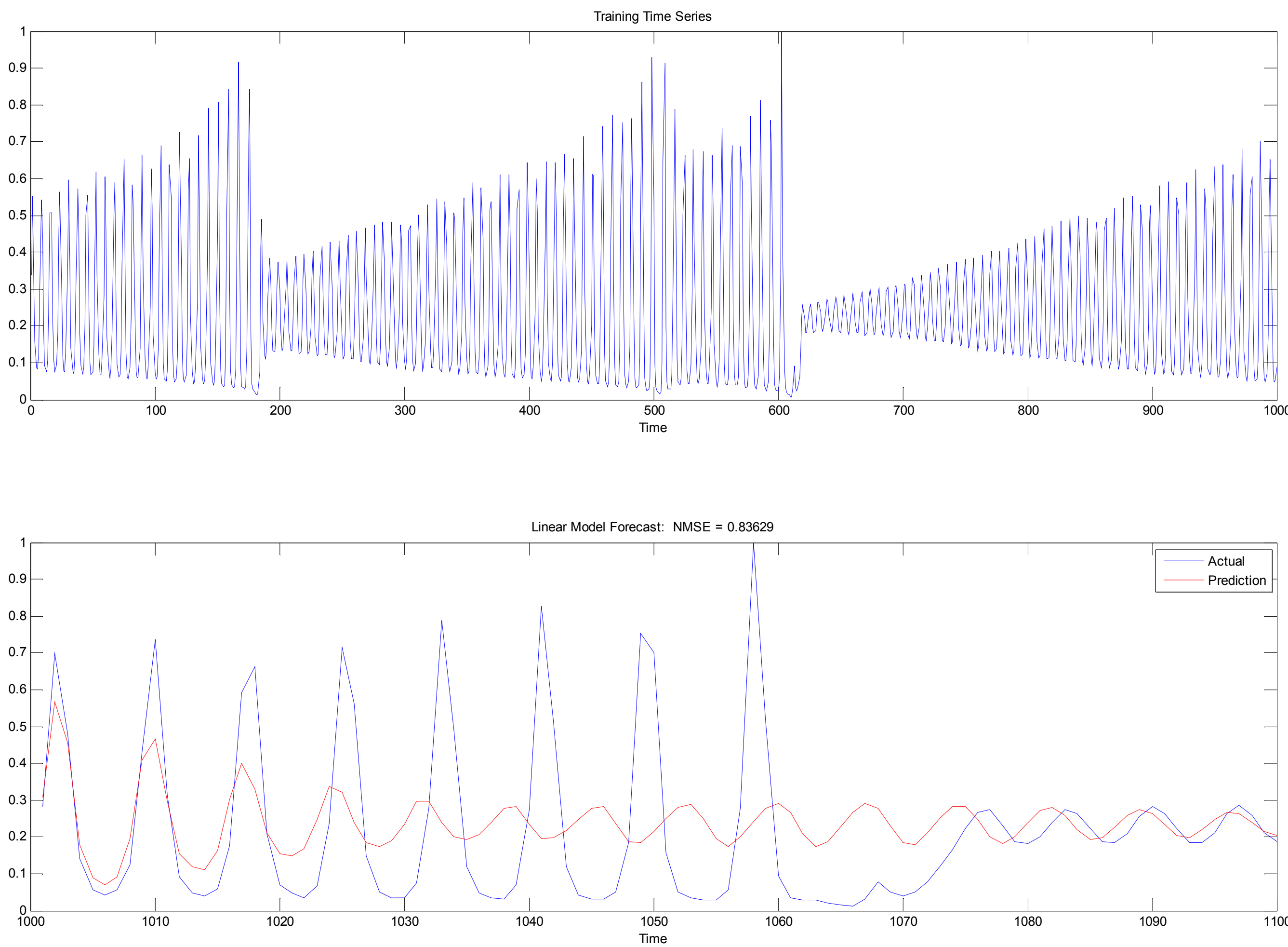
GP Trial B: NMSE = 0.16



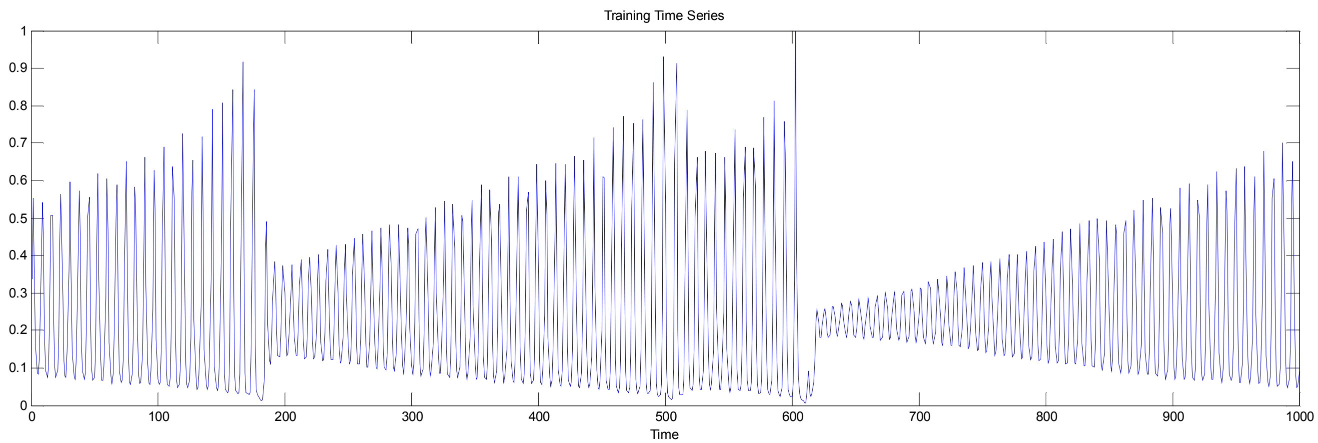


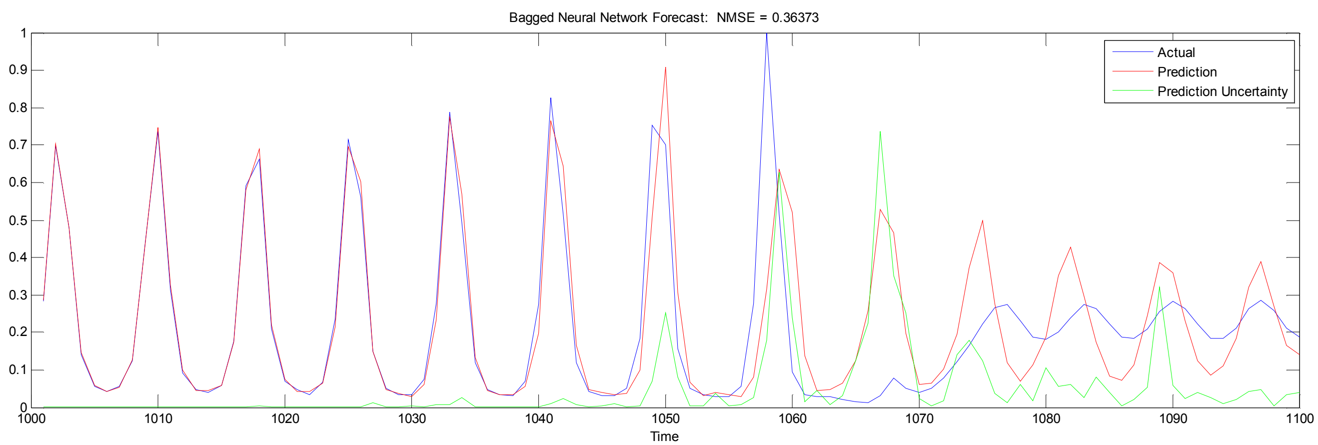
Very high accuracy and
corresponding low uncertainty

Linear Model: NMSE = 0.83



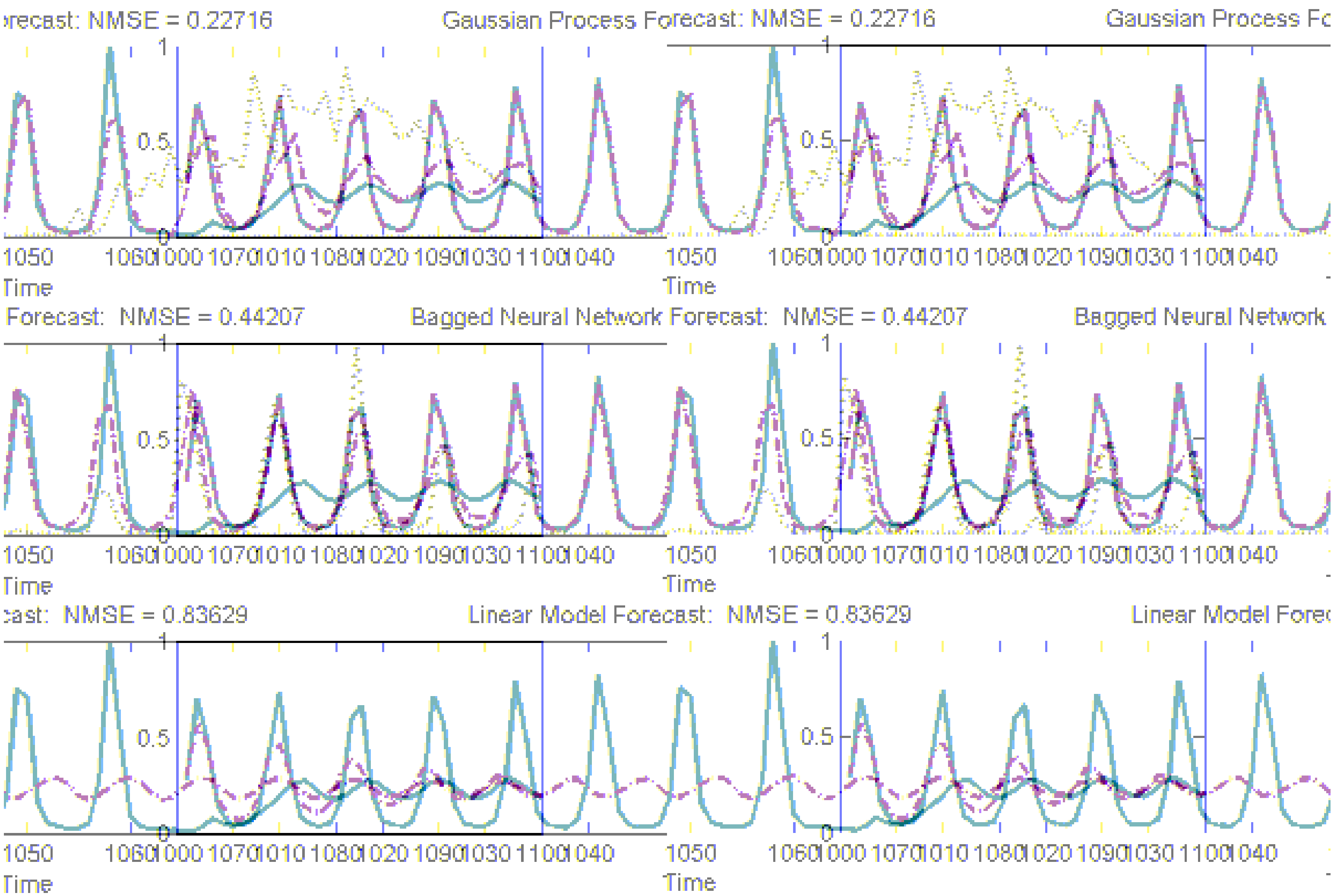
Bagged Neural Networks: NMSE = 0.37





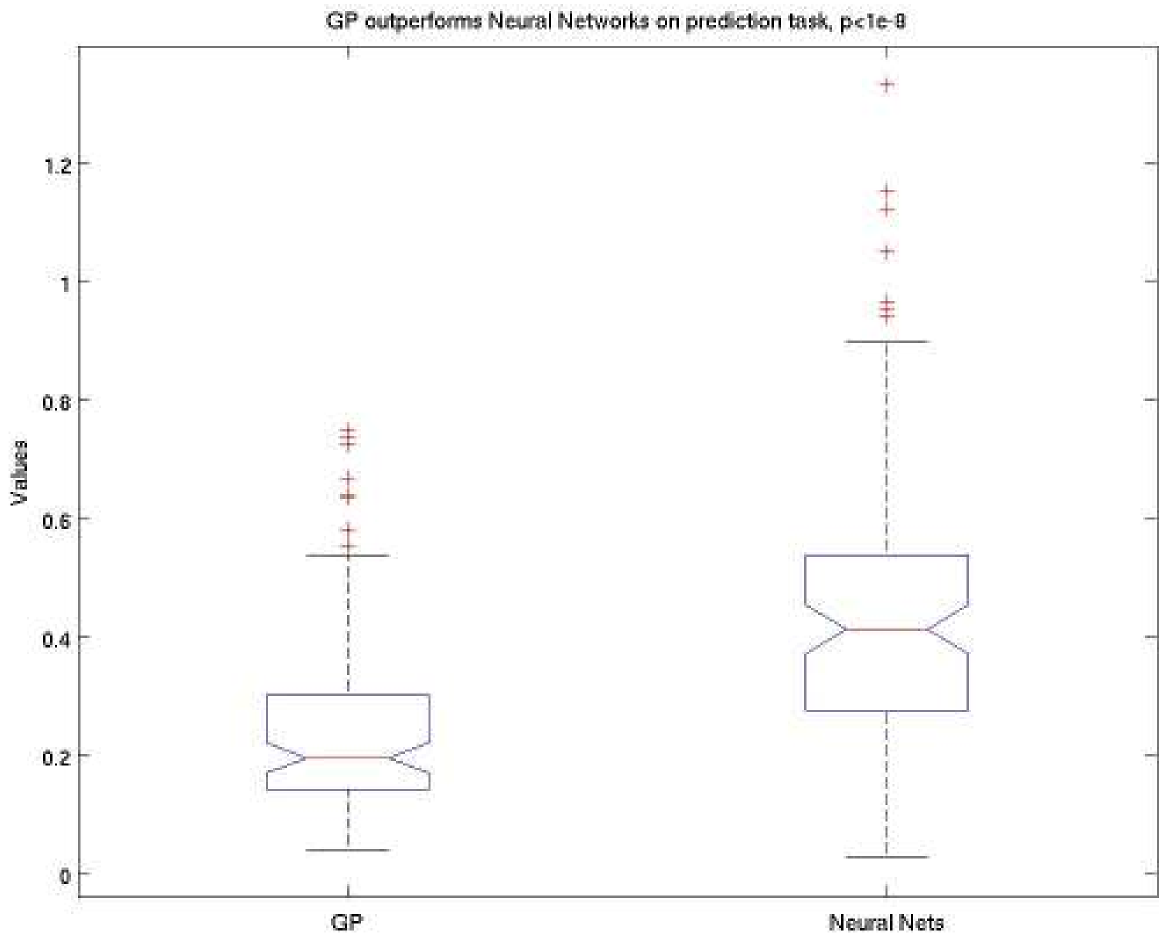
Note: although this model has a reasonable performance in this example, we found that it had significant run-to-run variability in performance.

Comparison



Gaussian Process Forecast: NMSE = 0.22716
Bagged Neural Network Forecast: NMSE = 0.44207
Linear Model Forecast: NMSE = 0.83629
Time

Statistical Comparison of GP's and Neural Networks



Results

- We have shown that we can make iterated forecasts and detect a precursor to the sudden drop in intensity using kernel methods.
- We can generate a meaningful measure of prediction certainty.
- This quantity seems to indicate substantial increases in uncertainty near the collapse.

Further Work

- Variability due to model uncertainty
- Significant testing with respect to forecast variability and quality of precursor detection.
- Analysis of forecast horizon.
- Test methods for use on Liquid Propulsion systems and ISS-CMG data sets.

References

- A. S. Weigend and N. Gershenfeld, “Time Series Prediction: Forecasting the Future and Understanding the Past”, 1994
- Gaussian Process Regression, J.S. Taylor, 2002
- Wikipedia